Accounting for Response Differences in Runoff Events of Different Magnitudes

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Abstract: Models that pass a series of estimated rainfall excess through a routing procedure of constant delay function neglect the fact that the magnitude of the flow event may have an impact on the catchment response. In this paper we demonstrate using flow data from two catchments that flow events of greater magnitude have a flashier response than smaller ones. The analysis of varying response according to event size is accomplished by utilising a two-parameter gamma function as a unit hydrograph and allowing the rainfall excess series to take any value less than or equal to the measured rainfall. Results indicate that the shape of the gamma function is event-dependent in the sense that larger events exhibit a flashier behaviour. In other words, a constant unit hydrograph will have problems in reproducing streamflow dynamics although the rainfall excess may adopt any reasonable values. Analysis of streamflow events reveals that the gamma distribution parameters could be related to the intensity of flow or rainfall excess.

Keywords: Rainfall; Streamflow; Unit hydrograph; Modelling

1. INTRODUCTION

Modelling rainfall-streamflow transformation requires computation procedures for runoff generation and flow routing within a catchment. The runoff generation component may simply be a loss module that determines rainfall excess, i.e. the part of rainfall that eventually becomes streamflow, or it can be a more physics-based model that describes evapotranspiration losses and soil and groundwater interactions, and produces the runoff input to the channel network. Flow routing may be based on simple delay functions, or the delay can be described with computation schemes simulating overland and channel flow toward the catchment outlet.

Concept of the unit hydrograph (UH) has commonly been used for determining the shape of the hydrograph resulting from rainfall excess [Sherman, 1932; Pilgrim and Cordery, 1992]. The unit hydrograph is defined as the total runoff response to a unit depth of rainfall excess produced by a storm of uniform intensity and specified duration. The instantaneous unit hydrograph (IUH) is the total runoff response to rainfall excess applied to the catchment over an infinitesimally short period. The assumptions behind the IUH are: 1) the linearity of streamflow response to rainfall excess; and 2) homogeneity in the spatial distribution of infiltration capacity, rainfall, and rainfall intensity. The linearity assumption assures applicability of convolution of the IUH with series of rainfall excess events. The second assumption implies that the IUH is a lumped representation of the catchment response.

Early applications of the UH consider direct runoff response to rainfall excess that primarily produces Hortonian surface runoff. In such applications baseflow needs to be separated first from the total streamflow hydrograph. Later models have been developed that compute IUH ordinates for the total streamflow response. Jakeman et al. [1990] used a transfer function which can represent both quick and slow flow components of the hydrograph.

One way to interpret the IUH is to consider it as a probability density function for water particle residence times within the catchment. For example, Rodríguez-Iturbe [1993] arrived at describing the IUH with a probability density function of the gamma distribution. Rodríguez-Iturbe and Valdés [1979] and Gupta et al. [1980] studied relationships between IUH parameters and catchment geomorphologic properties expressed through Horton's laws of drainage. These studies indicate that the geomorphologic instantaneous unit hydrograph (GIUH) performs better in large than in small catchments. Gupta et al. [1980] suggested that the implicit assumption of linearity of the rainfall excess-streamflow transformation is questionable for small catchments. Common to most applications is that the IUHs are constant with respect to time. This could be one reason limiting the applicability of IUH, since IUHs underlying streamflow events of different magnitudes may not be identical.

Chen and Singh [1986] recounted that early studies recognised variability of the UH with the intensity of rainfall excess, but few studies have implemented variable UHs in hydrological modelling. Chen and Singh [1986] extended the variable UH of Ding [1974] and embedded a nonlinear relationship between rainfall excess and direct runoff in their IUH. Georgakakos and Kabouris [1989] introduced a GIUH that accounted for both surface and subsurface runoff and that was time-variable on an event basis. Results from variable IUH applications suggest that it can be a promising way to characterise response differences for streamflow events of different magnitudes.

The goals of this study are to

1) investigate one- and two-hour rainfall-streamflow time series to identify variable UHs for events of different magnitudes

2) demonstrate deficiencies of the time-invariant (nonvariable) UH using a two-parameter gamma distribution

3) examine methods to form variable UHs as a function of streamflow or rainfall excess intensity

2. SITES AND DATA

2.1 Siuntio, Finland

Streamflow and meteorological data were available from a small research basin of the Finnish Environment Institute located in southern Finland (Rudbäck, 0.18 km^2, 60.2 N, 24.1 E). The catchment is covered by a mature forest stand dominated by Norway spruce. Elevation ranges from 34 to 65 metres above mean sea level. Bedrock is exposed on the hilltops and soils are composed of silty and sandy moraines with an average depth of 1-2 metres to the bedrock. More details on the site are published in Lepistö [1994] and Lepistö and Kivinen [1997].

The climate in Siuntio is temperate with cold, wet winters and precipitation is typically of a relatively low intensity including approximately 30% snowfall annually. Mean annual precipitation, uncorrected for wind effects was 700 mm during 1991-96. Mean monthly temperatures in February and July are –2°C and 16°C, respectively.

In 1996 a measurement campaign was initiated to provide meteorological, snow and streamflow data for calibration and validation of hydrological models. These data from 1996 to 2001 include hourly records of precipitation, air temperature, relative humidity, wind speed, downward and reflected short-wave radiation, and long-wave radiation from an open site next to the catchment. Catchment data include hourly streamflow, and weekly measurements of snow depth and snow water equivalent, depth to the groundwater table, and throughfall beneath the canopy.

2.2 Andrews Watershed 2, USA

A precipitation-streamflow dataset was available from the H. J. Andrews Experimental Forest of the Willamette National Forest, Oregon, USA. Quarter-hourly streamflow and precipitation measurements were from watershed 2 (0.63 km^2 44.2 N, 122.2 W), and daily air temperature time series was from the primary meteorological station. Elevation ranges from 548 m to 1070 m. Watershed 2 is completely forested with Douglas-fir and western hemlock as the dominating tree species. Loam and clay loam soils occur in ridgetop and steep slope positions. Stone content ranges from 35 to 50%, generally increasing on south-facing slopes. Depth to weathered parent material is usually over 1 m. More detailed information about the site are published in Hawk and Dyrness [1975].

The maritime climate has wet, mild winters and dry, cool summers. At the primary meteorological station at 430 m elevation, mean monthly temperature ranges from near 1°C in January to 18°C in July. Average annual precipitation varies with elevation from about 2300 mm at the base to over 3550 mm at upper elevations, falling mainly in November through March. Rain predominates at low elevations; snow is more common at higher elevations. Highest streamflow occurs generally in

November through February during rain-on-snow events.

The precipitation and streamflow data covered the period from 1958 to 1990. In addition, snow depth and meteorological data from the central climatic station at an elevation of 1017 m were used in the calibration of a snow model.

3. METHODS

The time-convolution of rainfall excess u with an instantaneous unit hydrograph h determines streamflow Q as

$$Q(t) = \int_0^t h(t-s)u(s)ds \qquad (1)$$

where t is time. In this study, the IUH is selected to be a two-parameter gamma distribution

$$h(t) = \frac{1}{\beta^{\alpha}\Gamma(\alpha)} t^{\alpha-1} e^{-t/\beta} \qquad (2)$$

where α and β are parameters, and Γ is the gamma function. One- and two-hour UHs were computed via integration of $h(t)$ in one- and two-hour segments, respectively.

Rodríguez-Iturbe [1993] suggested relationships between parameters of the gamma distribution and catchment geomorphology. According to his analysis α depended merely on geomorphologic properties, whereas β depended on both geomorphology and streamflow velocity. Following these results, a variable IUH was formed by assigning α with a constant value and relating β to the intensity of rainfall excess or streamflow:

$$\beta = \frac{a}{b+x} \qquad (3)$$

where a and b are variable UH parameters, and x is the intensity of streamflow or rainfall excess.

4. RESULTS

4.1 Selection of Rainfall-Streamflow Events

Precipitation-streamflow series were screened to select individual events for the UH case study. The selection criteria were: 1) the events should not be affected by snow accumulation or snowmelt, 2) the events should cover a large range of peak-flow intensities, and 3) considerable portion of flow recession should occur under rainless conditions. Time step of the precipitation-streamflow series was 1 hour for Siuntio data and 2 hours for Andrews WS2 data. A longer time step was selected for the rainier Andrews catchment in order to restrict the number of rainfall excess values to be optimised.

For Siuntio data streamflow events occurring during snow accumulation or melt were disregarded following the modelling results of Koivusalo and Kokkonen [2000]. Since the measurement period in Siuntio was less than 5 years and the number of large streamflow events was small, events with low flow intensities (less than 0.2 mm/h) were also included in the study. Rainfall during flow recessions occurred only rarely for most of the Siuntio events. The total number of selected events was 13 for Siuntio. Table 1 lists flow and rainfall volumes and maximum intensities for the largest event and the smallest event in Siuntio.

Streamflow events in WS2 in the Andrews Experimental Forest were more difficult to select, because the higher elevated portion of the catchment was presumably snow affected in most winters. The occurrence of snow accumulation and melt was estimated using a degree-day snowmelt model [e.g., Kuusisto, 1984] which used daily precipitation and air temperature time series as an input. The degree-day model was first calibrated against snow depth data from the central meteorological station, which receives more snowfall than WS2 due to its higher elevation. Figure 1 shows measured and simulated snow depths for the period from November 1997 to May 1999. Subsequently, the calibrated snow model was used to simulate snow water equivalent in the WS2 with meteorological input data from the primary meteorological station. Many of the highest streamflow events in WS2 occurred during rain-on-snow conditions and were thus omitted in the selection of events to be studied. However, some large events, which had clearly higher rainfall intensities compared with predicted snowmelt intensities, were selected. Length of the record was sufficiently long to select only events with peak flow intensities greater than 1 mm per 2 hours. The total number of selected events was 19 for WS2. Table 1 lists flow and rainfall characteristics for the largest event and the smallest event in WS2.

4.2 Nonvariable Unit Hydrograph

Gamma distribution parameters and rainfall excess time series were optimised concurrently for each event separately. The rainfall excess was taken as a constant of the measured rainfall with the restriction that it was not allowed to exceed the measured rainfall at any time step. The idea was to examine if there existed a reasonable rainfall excess series and a nonvariable UH that would reproduce the measured streamflow adequately. The optimisation was carried out using the shuffled complex evolution method (SCE-UA) of Duan et al. [1992, and 1993]. The sum of squared error between measured and calculated streamflow was used as the objective criterion.

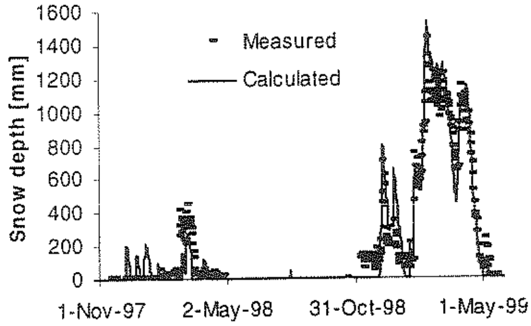

Figure 1. Measured and calculated snow depth at the central meteorological station during 1997-99.

Table 1. Times of occurrence, and sums and maximum values of rainfall (*P*) and streamflow (*Q*) for the largest and smallest events in Siuntio and Andrews WS2.

	Time	Total *P* mm	Total *Q* mm	Max *P* mm/h	Max *Q* mm/h
Siuntio	4-8 Nov 00	50.6	36.8	6.6	1.4
Siuntio	17-19 Jun 98	15.1	1.4	2.1	0.06
WS2	8-12 Jan 89	153.5	104.1	8.8	3.3
WS2	2-6 Feb 74	59.6	30.1	4.6	0.78

Figure 2 shows one-hour UHs for 13 events in Siuntio. UHs are plotted in descending order of the computed event peak-flow intensities. One can detect that the UHs for streamflow events with higher peak intensities indicate a flashier response. Notable exceptions are the flashy UHs for events with very small streamflow magnitudes. Further examination of the results reveals that for high streamflow events the rainfall excess is constrained by the measured rainfall, but for the very small events rainfall excess is always far less than the measured rainfall and is thus practically unconstrained by the rainfall. Therefore, the optimisation is not likely to yield plausible estimates of rainfall excess and UH parameters for such events.

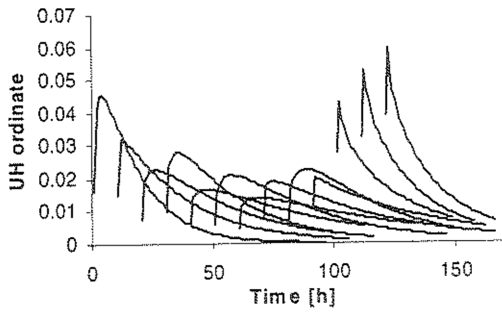

Figure 2. One-hour UHs for the events in Siuntio. UHs are plotted from left to right in descending order of peak-flow intensity.

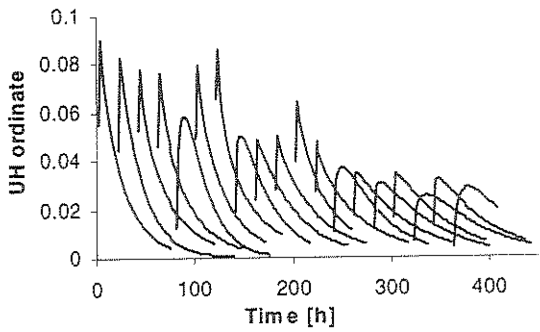

Figure 3. Two-hour UHs for the events in Andrews WS2. UHs are plotted from left to right in descending order of peak-flow intensity.

Figure 3 shows the two-hour UHs for streamflow events in Andrews WS2. Again, a clear trend can be detected in the UHs, which show a weakening of response dynamics with peak flow intensity. Since Andrews data included plenty of high-intensity streamflow events, very small events were not included in the computations. Consequently, the UHs from Andrews data do not show inconsistency similar to the small event results in Siuntio.

Figure 4 shows that the nonvariable UHs cannot reproduce the observed flow recession for the largest streamflow events in Siuntio and Andrews WS2, even with an optimised rainfall excess series (note: another objective function could yield a good fit to the recession part of the hydrograph by compromising fit to the peak). This result holds only for the largest flow events, for small and medium-size events optimisation yields rainfall excess series and UH parameters that reproduce adequately the measured streamflow.

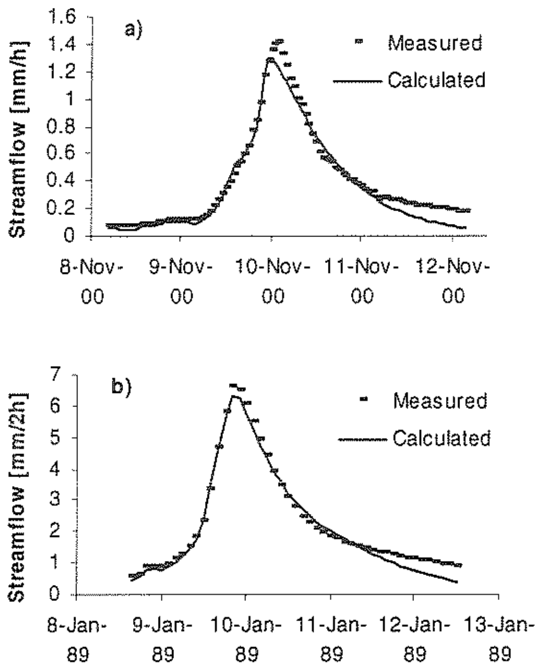

Figure 4. Largest measured and computed streamflow events in Siuntio (a) and Andrews WS2 (b) using nonvariable UHs.

4.3 Variable Unit Hydrograph

Relationships between nonvariable UHs and peak intensities of streamflow or rainfall excess are examined to reveal the extent to which UH parameters depend on the event size. Table 2 shows regressions between gamma distribution parameters (α and β) and calculated peak streamflow (and rainfall excess) intensities. The results indicate that the slopes of the regression lines are significant at 5% risk level only for β, whereas α tends to be constant regardless of the event size. This result is in line with the GIUH model proposed by Rodríguez-Iturbe [1993], where α depends only on catchment geomorphologic properties and β depends on both geomorphology and flow characteristics.

Figure 5 presents optimisation results for the two highest streamflow events from Siuntio and Andrews WS2 using the variable UH. The β parameter of the variable UH is related in turn both to rainfall excess and simulated streamflow according to equation (3). The graphs show that unlike in the case of the nonvariable UH, one can identify rainfall excess time series that reproduces adequately measured flow characteristics. The results are similar when the gamma distribution parameter β is related either to computed flow intensity or to computed rainfall excess.

Table 2. Regressions between UH parameters and calculated peak streamflow (and rainfall excess) intensity for events in Siuntio and in WS2.

Against Flow Q_{max} [mm/h]	Regression Equation	Slope p-value
Siuntio data	$\alpha = 1.15 + 0.014\ Q_{max}$	0.88
Siuntio data	$\beta = 42.4 - 22.4\ Q_{max}$	0.03
Andrews data	$\alpha = 1.19 - 0.076\ Q_{max}$	0.16
Andrews data	$\beta = 54.2 - 9.20\ Q_{max}$	0.00
Against Rainfall Excess u_{max} [mm/h]	Regression Equation	Slope p-value
Siuntio data	$\alpha = 1.15 + 0.002\ u_{max}$	0.92
Siuntio data	$\beta = 49.6 - 4.77\ u_{max}$	0.02
Andrews data	$\alpha = 1.21 - 0.027\ u_{max}$	0.34
Andrews data	$\beta = 63.6 - 4.48\ u_{max}$	0.00

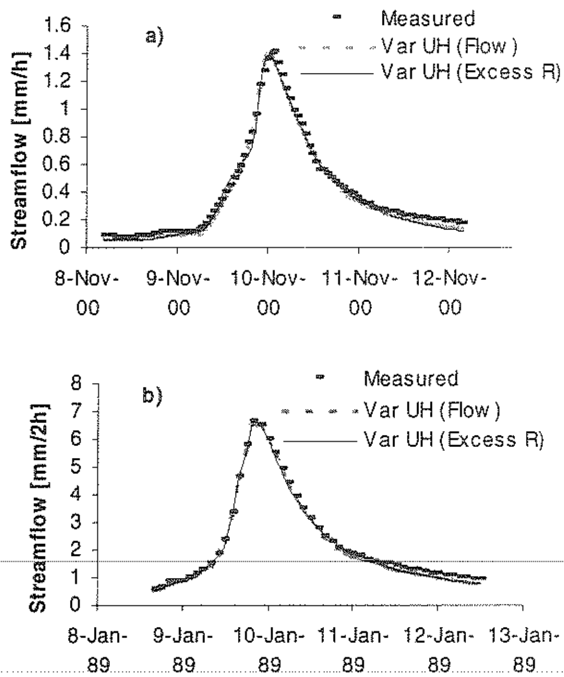

Figure 5. Largest measured and computed runoff events in Siuntio (a) and Andrews WS2 (b) using variable UHs.

5. CONCLUSIONS

The results indicate that given any reasonable time series of rainfall excess, reproduction of high-intensity streamflow hydrographs was poor using a nonvariable UH. For events of different magnitudes, the UH characteristics in terms of fitted gamma distributions show a clear decrease in response dynamics with the peak flow intensity.

Analysis of streamflow events revealed that the gamma distribution parameter α was independent of the event size, and the parameter β could be related to the intensity of flow or rainfall excess. These results are consistent with the GIUH model of Rodríguez-Iturbe [1993], where β is related to streamflow velocity. The above relationship led to a formulation of a variable UH, for which it was possible to find a reasonable rainfall excess series that reproduced the measured streamflow hydrograph adequately.

Future work should be directed towards identifying which one of the variables, flow intensity or rainfall excess intensity, is a better index for the UH variability. Identification of rainfall excess series provides an opportunity for inverse rainfall-runoff modelling, i.e., calibrating runoff generation procedures against rainfall excess time series derived from measured runoff and a variable UH.

6. ACKNOWLEDGEMENTS

This study was funded by the Academy of Finland project "Predicting impacts of land use changes on catchment hydrological processes". We greatly appreciate support provided by the head of the project, Prof. Pertti Vakkilainen. Andrews data sets were provided by the Forest Science Data Bank, a partnership between the Department of Forest Science, Oregon State University, and the U.S. Forest Service Pacific Northwest Research Station, Corvallis, Oregon. Significant funding for these data was provided by the National Science Foundation Long-Term Ecological Research program (NSF Grant numbers BSR-90-11663 and DEB-96-32921).

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